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1985 J. Phys. A: Math. Gen. 18 L369

(http://iopscience.iop.org/0305-4470/18/7/007)

J. Phys. A: Math. Gen. 18 (1985) L369-L371. Printed in Great Britain

LETTER TO THE EDITOR

Critical spin-wave dynamics for two-dimensional Sierpinski-type fractal

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Received 10 December 1984

Abstract. The dynamic critical exponent z for a system of nearest-neighbour interacting Heisenberg spins on a two-dimensional Sierpinski-type fractal is obtained by applying a real space renormalisation group transformation of the equations of motion.

In recent years there has been considerable interest in studying extensively the Sierpinski gasket type fractal structures as models for the backbone of the infinite percolating cluster (Harris and Stinchcombe 1983, Gefen *et al* 1984). We consider here a system of nearest-neighbour interacting Heisenberg spins on a two-dimensional ($d=2$) Sierpinski-type fractal (STF) (Hilfer and Blumen 1984) at zero temperature. The fractal dimensionality (d_{f}) of the STF (see figure 1) is given by

$$d_{\mathrm{f}} = \ln 6/\ln 3 \ (\approx 1.631). \tag{1}$$

It is interesting to note that this value of d_{f} agrees well with a recent estimate of the backbone fractal dimensionality ($d_{\mathrm{f}}^{\mathrm{BB}} = 1.62 \pm 0.03$, $d=2$) (Herrmann and Stanley 1984). One expects, therefore, the STF to provide the essential features of the low frequency (critical) Heisenberg spin dynamics on the backbone of the infinite cluster embedded in two dimensions at the percolation threshold.

The equation of motion for the transverse spin component S_i, at site i, is

$$\alpha S_i = \sum_j \phi_{ij}(S_i - S_j) \tag{2}$$

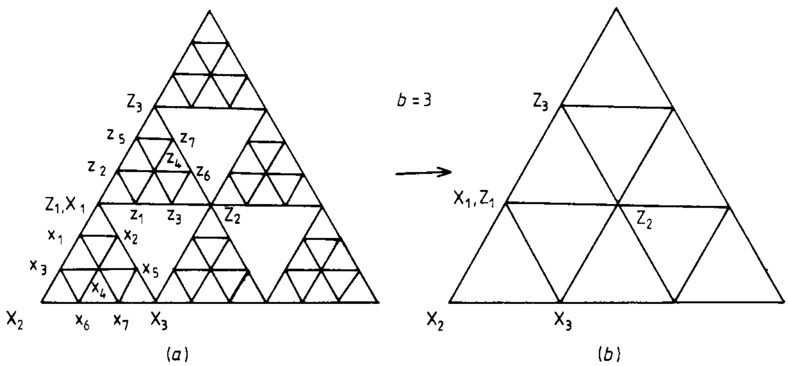

Figure 1. Transformation of the Sierpinski-type fractal lattice by a scale factor $b=3$.

where $\alpha = \omega/J$, ω and J are respectively the characteristic frequency and the exchange, $\phi_{ij} = 1$ if i, j are nearest neighbours and zero otherwise.

We now illustrate the decimation transformation with scale factor $b = 3$ starting from the set of equations of motion (2) for the sites $\{x_i\}$ of the lowest left-hand triangle of figure 1(a):

$$
\begin{aligned}
(4-\alpha)S_{x_1} &= S_{x_2} + S_{x_3} + S_{x_4} + S_{X_1} \\
(4-\alpha)S_{x_2} &= S_{x_1} + S_{x_4} + S_{x_5} + S_{X_1} \\
(4-\alpha)S_{x_3} &= S_{x_1} + S_{x_4} + S_{x_6} + S_{X_2} \\
(6-\alpha)S_{x_4} &= S_{x_1} + S_{x_2} + S_{x_3} + S_{x_5} + S_{x_6} + S_{x_7} \\
(4-\alpha)S_{x_5} &= S_{x_2} + S_{x_4} + S_{x_7} + S_{X_3} \\
(4-\alpha)S_{x_6} &= S_{x_3} + S_{x_4} + S_{x_7} + S_{X_2} \\
(4-\alpha)S_{x_7} &= S_{x_4} + S_{x_5} + S_{x_6} + S_{X_3}.
\end{aligned}
\tag{3}
$$

The corresponding equation for S_{X_1} (or S_{Z_1}) is

$$(4-\alpha)S_{X_1} = S_{x_1} + S_{x_2} + S_{z_1} + S_{z_2}. \tag{4}$$

Using equations (3), one obtains

$$S_{x_1} + S_{x_2} = a\ (S_{X_2} + S_{X_3}) + bS_{X_1} \tag{5}$$

where $a = (\alpha^2 - 14\alpha + 42)/c$, $b = 2(13\alpha^2 - 49\alpha + 48 - \alpha^3)/c$, $c = (3-\alpha)(13\alpha^2 - 46\alpha + 30 - \alpha^3)$, and similarly,

$$S_{z_1} + S_{z_2} = a(S_{Z_2} + S_{Z_3}) + bS_{Z_1}, \qquad (S_{Z_1} \neq S_{X_1}). \tag{6}$$

A new relation where S_{X_1} is a function of S_{X_2}, S_{X_3} and S_{Z_2}, S_{Z_3} can now be extracted from equations (4), (5) and (6). This relation can be reduced to a form exactly similar to that of the first equation (3) with the renormalised dynamic parameter

$$\alpha' = \alpha(\alpha^2 - 14\alpha + 42)^{-1}(\alpha^4 - 20\alpha^3 + 145\alpha^2 - 452\alpha + 510). \tag{7}$$

Linearising this recursion relation (7) for the characteristic frequency around the fixed point $\alpha^* = 0$, one obtains the dynamic exponent z (Harris and Stinchcombe 1983)

$$z = \ln\,(85/7)/\ln 3 \qquad (\approx 2.273).$$

It may be mentioned that one can also extract the spectral dimension ($\tilde{d}$) of the STF of figure 1 from relation (7). For this one has to consider a system of identical masses m located at the sites of the fractal lattice, joined by springs of strength K, i.e. to replace α's in relation (7) by $m\omega^2/K$. One then obtains following the scaling arguments of Rammal and Toulouse (1983)

$$\tilde{d} = 2\ln 6/\ln\,(85/7) \qquad (\approx 1.435).$$

It is to be noted that this value of d does not agree with that of Hilfer and Blumen (1984), which is

$$\tilde{d} = 2\ln 6/\ln\,(90/7) \qquad (\approx 1.403).$$

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